MATH Literacy Toolkit

Intro to Logarithms Toolkit

SNIPPETS FROM THE LESSON

ProActive Ed

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State Standards

☑ TEKS A2.2C

☑ TEKS A2.5A-C

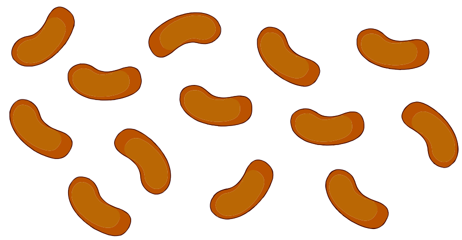
NCTM Process Standards

Problem Solving	Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts. Apply and adapt a variety of appropriate strategies to solve problems. Monitor and reflect on the process of mathematical problem solving.
Reasoning and Proof	Make and investigate mathematical conjectures. Select and use various types of reasoning and methods of proof.
Communication	Organize and consolidate student mathematical thinking in written and verbal communication. Communicate mathematical thinking clearly to peers, teachers, and others. Use the language of mathematics to express mathematical ideas precisely
Connections	Recognize and use connections among mathematical ideas. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. Recognize and apply mathematics in contexts outside of mathematics.
Representations	Create and use representations to organize, record, and communicate mathematical ideas. Select, apply, and translate among mathematical representations to solve problems. Use representations to model and interpret physical, social, and mathematical phenomena.

Learning Objectives

Students develop a conceptual understanding of the relationship between logarithmic functions and their exponential inverses. They then apply that understanding to rewrite these corresponding equation, analyze the effects on the key attributes on the graphs of logarithmic functions, model real-world situations, solve problems involving logarithmic and exponential inverses, and determine the reasonableness of their solutions.

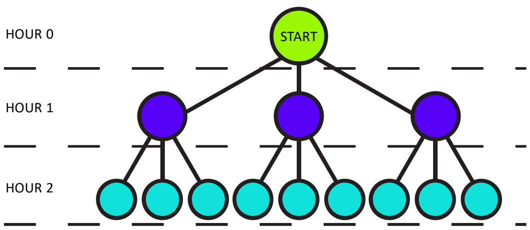
Toolkit Materials



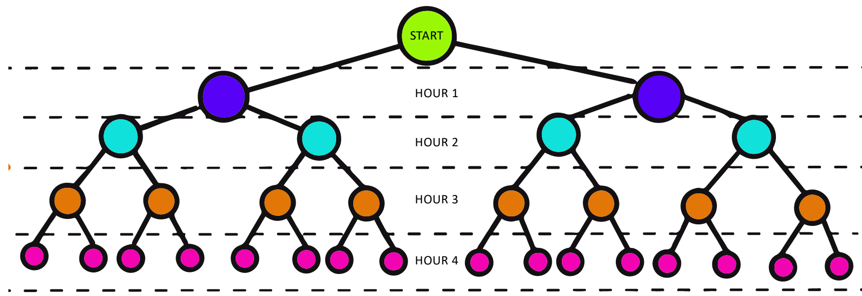
Concrete Representations
☑ Bag of Beans (at least 81 beans)

Blackline Masters
☑ Base 2 Mitosis Mat
☑ Base 3 Mitosis Mat

Not Included
❒ Scissors



CELL MITOSIS



Literacy Guide

	Academic Discourse	Engage in conversations about the big ideas
	Conceptual Understanding	Explore the math using hands-on materials
	Informational Text	Read and write about concepts and problem solving strategies
	S.T.E.A.M. Connections	Investigate science, technology, engineering and art topics using the math
	Technical Writing	Present and write about the S.T.E.A.M. Connections

Recommended Intervention Toolkit
Evaluating Linear and Exponential Functions Toolkit

Recommended Acceleration Toolkit
Building Radical, Rational, and Composite Functions Toolkit

Teacher Tips

Anchor 1: Academic Discourse

- ☑ Use games like a scavenger hunt to help students see the mathematics in the universe that surrounds them.
- ☑ Connect prior learning to make real-world connections to the learning goal.
- ☑ Reduce the barrier of academic vocabulary by focusing on big ideas and real world representations.

Anchor 2: Conceptual Understanding

- ☑ Use concrete realia or virtual manipulatives to represent the learning objective.
- ☑ Use hand-on manipulatives and student created pictures before transitioning to abstract concepts and standard algorithms.
- ☑ Use laboratory procedures that follow a constructivist approach to investigate the topic and learn key concepts.
- ☑ Communicate learning experiences through academic dialogue
- ☑ Write expository pieces to demonstrate conceptual understanding of the learning topic.

Anchor 3: Informational Text

- ☑ Use informational text to investigate the topic and learn key terms.
- ☑ Use reading strategies like previewing, chunking, annotating, and text dependent questioning to help students process the density text.
- ☑ Encourage reading and English teachers to utilize informational text about mathematics in their classroom settings.
- ☑ Communicate learning experiences through academic dialogue
- ☑ Write expository pieces to analyze the concepts and strategies presented in the text.

Anchor 4: S.T.E.A.M. Connections

- ☑ Use research, context clues, and access student schema to comprehend the given scenario
- ☑ Investigate invented strategies and standard algorithms to determine potential successes and failures.
- ☑ Design a prototype that satisfies the criteria outlined in the project before creating the final product.
- ☑ Collaborate with others to share strategies, critique reasoning, and justify methods.

Anchor 5: Technical Writing

- ☑ Write paragraphs that summarize the S.T.E.A.M. scenario. Be sure to include the criteria and scoring guide.
- ☑ Write paragraphs that describes the steps that will be used to address the scenario. Be careful to use numbers with a description of the role those numbers play in those steps.
- ☑ Write paragraphs that incorporates the steps used to address the scenario into actual calculations that include graphs, charts, diagrams and other representations as deemed appropriate
- ☑ Write paragraphs that investigate alternative problem solving strategies as a means for verifying the accuracy and validity of solutions
- ☑ Write paragraphs that reflect on strengths, misconceptions, and potential future applications of the concepts that were addressed and the strategies that were used.

Math Conversations

SETUP THE GAME

In math, we always want a way to "undo" something we've done. So if we have addition, we want to be able to have subtraction to "undo" it. If we have multiplication, we want to have division to "undo" it. If we have exponents, like we just learned about, we want something called logarithms to "undo" the exponents.

In real life we also have actions that "undo" other actions. For example, your car can drive forwards, but you can also put it in reverse and drive backwards - "undoing" your progress forwards.

Stop and write an example of an action that "undoes" something else.

For the following scavenger hunt, you are going to come up with examples of actions or items that "undo" something in the real world.

PLAY THE GAME - PART 1

Step 1: Set a timer for 5 minutes

Step 2: Think of as many examples of actions or items that "undo" something in the real world.

Step 3: Keep track of these examples in the table below. List what they "undo" next to them.

Example	What does it "undo"?

What's Old?

In math (and in real life) there are things that "undo" each other. In math, -2 "undoes" +2, and 1/5 "undoes" 5. In real life, turning a light off "undoes" turning the light on, or erasing your pencil marks "undoes" your writing.

What's New?

We are now going to learn about a special type of "undoing" in math. There's an entire function in math that is designed to "undo" something else - it is called a **logarithm** and is designed to undo exponents.

Review: A number with an exponent is something like

$$2^2$$
$$5^3$$
$$10^{21}$$

The big number at the bottom is called the base, and the number above it is called the exponent.

$$(\text{base})^{\text{exponent}}$$

The exponent tells us the number of times we multiply the base times itself.

$$2^2 = \underbrace{2 \cdot 2}_{\text{2 times}} = 4$$

$$5^3 = \underbrace{5 \cdot 5 \cdot 5}_{\text{3 times}} = 125$$

$$10^{21} = \underbrace{10 \cdot 10 \cdot ..... \cdot 10 \cdot 10}_{\text{21 times}} = 1000000000000000000000$$

A logarithm is a function that takes the base number and the "answer" (like 4 or 125) to give you what the exponent was. A logarithm is written as "log" with a base.

$$\log_{\text{base}}(\text{"answer"}) = \text{exponent}$$

Math Explorations

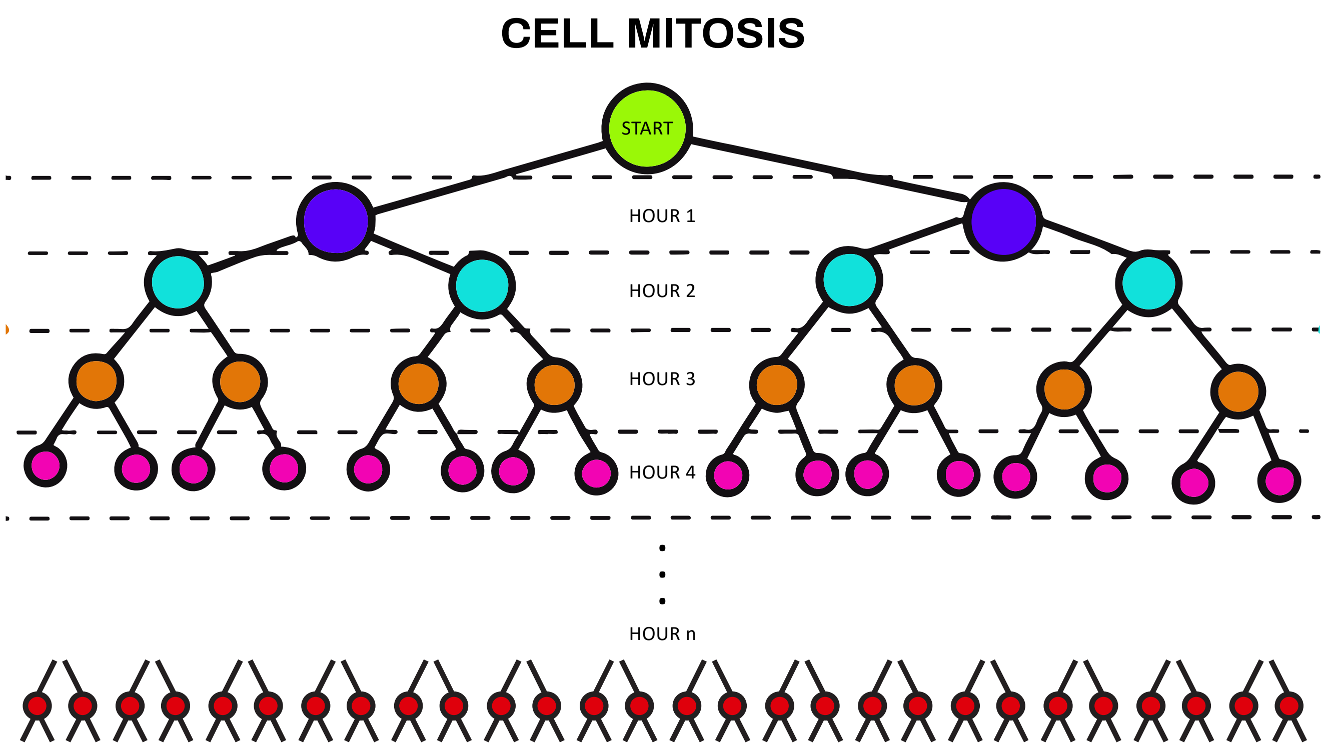
Let's revisit your dividing cells from before. You now know how to figure out the # of Cells (or really, beans) at any hour, but what if you were given a # of Cells (or beans) and asked to find the hour you were at? Or what if your cells were a little mutated and divided into three cells each time instead of two? How would you answer those questions?
In the following activities, you will use your dividing cells to model "undoing" an exponent by starting with a final answer and discovering where you started.

Part 1: The original cells, just backwards

Step 1: Gather your dry beans and a pencil.

Stop and Think: If you were given how many cells you had, could you use that information to find out the hour you were at?

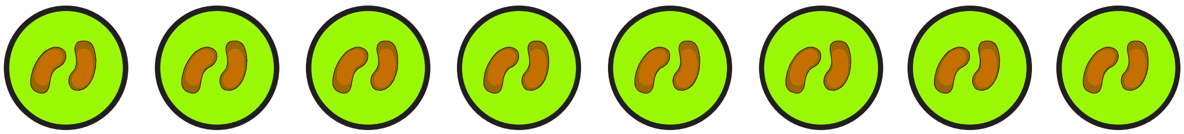
Step 2: Consider the cell division below:



Originally, it's the same cell mitosis we saw before. But now we have Hour n, with lots of cells. How do we figure out what n is?

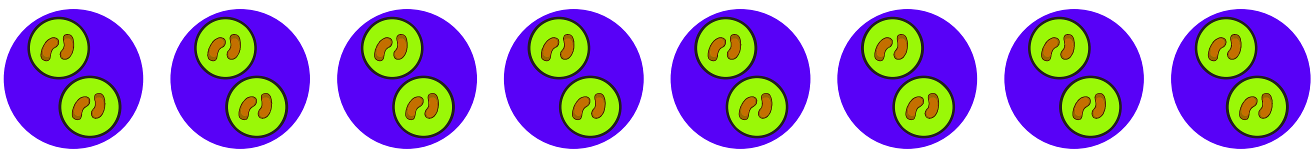
Step 3: Put one dry bean on each of the cells for Hour n.

Step 4: Group the beans into groups of 2.



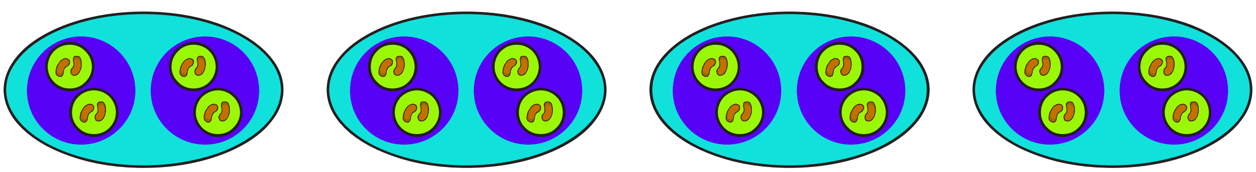
If you let each group represent a "parent" cell, you now have the # of Cells at Hour n-1.

Step 5: Group the groups of beans into 2.



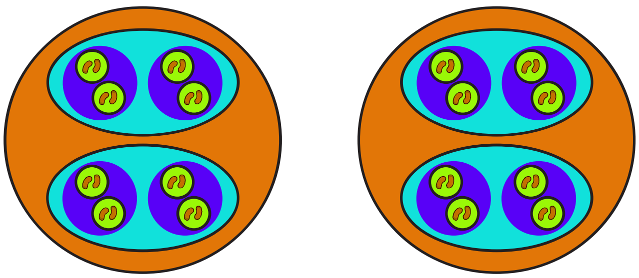
This is the # of Cells at Hour n-2.

Step 6: Group these groups of beans into 2.



This is the # of Cells at Hour n-3.

Step 7: Group these groups of beans into 2.



This is the # of Cells at Hour n-4.

Stop and Think: If there are two "cells", what Hour are we at?

Step 8: At Hour n-4, there are 2 cells. If we look at our table, we see that when there are 2 cells, it is Hour 1.

Hour	Beans
0	1
1	2
2	4
3	8

Step 9: Thus, Hour n-4 is really Hour 1. That means that Hour n is really Hour 5.

Step 10: Count your beans from Hour n. (You should have 32 beans.) Earlier, we found showed that the # of Beans at Hour n is equal to 2^n. So at Hour 5, there should be 2^5 beans, or 32 beans.

What if we wanted to solve this problem mathematically? We would ask what n is when $32 = 2^n$. How do we find out the answer though? This is where logarithms come in. Remember when we said that a logarithm is a function that "undoes" exponents? If we have a number:

$$(\text{base})^{\text{exponent}} = \text{answer}$$

then the logarithm says:

$$\log_{\text{base}}(\text{"answer"}) = \text{exponent}$$

So,

$$2^n = 32 \quad \xleftrightarrow{\text{"undo"}} \quad \log_2(32) = n$$

Using our dry beans, we determined that n =5, so:

$$\log_2(32) = 5$$

We can confirm this with a calculator and we get:

Reading & Writing Instructions

Identify the Craft and Structure
- Find and highlight definitions for variables, inequalities, and equations.
- Write your own definitions in the margins.
- Share your definitions with a partner.
- Read the passage and stop at every word you don't know. Place a dot above the words and keep reading.
- Compare your dotted words with a partner and try to figure out what they mean.
- Write your meanings in the margin
- Reread the passage using your definitions.

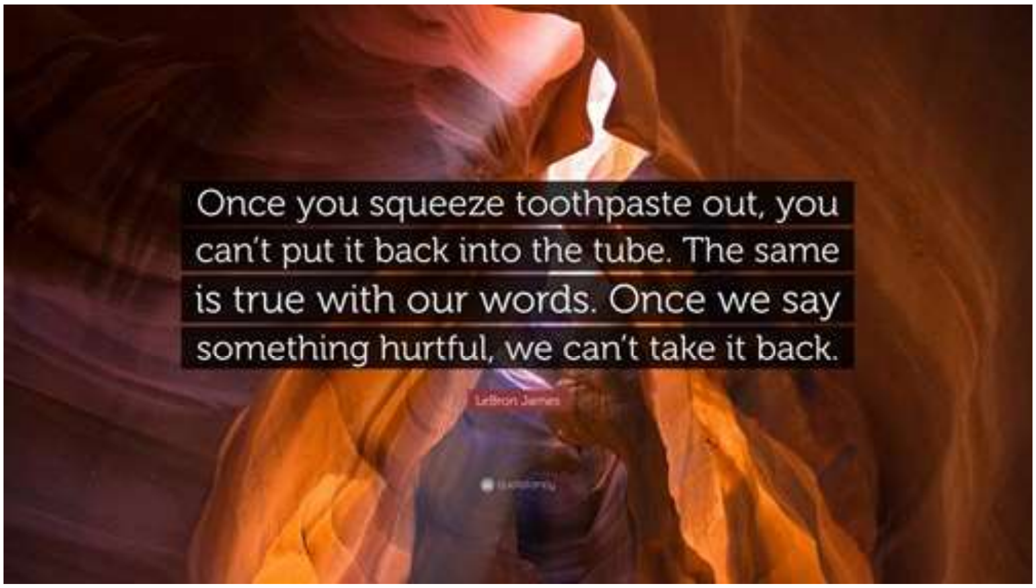
Find the Key Ideas and Details
- What is the text about?
- What are logarithms and what do they "do"?
- What kind of equations can we use logarithms to solve?
- How do we solve equations involving exponents and logarithms?

Integrate Your Knowledge and Ideas
- Give examples of exponents and/or logarithms.

Write: Letter to Your Parent or Guardian
- Write a summary explaining what you know about exponents and logarithms .
- Provide your own examples of uses of exponents and logarithms.
- Use complete sentences, diagrams and pictures as needed.
- Include at least 2 vocabulary words in your writing.

Undo, Redo

Have you ever been told that in life, you can't always take back what you've said or done?



(Even LeBron James said it!)

I have good news for you. In math, we can always "take back" what we've done. If we've added, we can subtract. If we've multiplied, we can divide. And if we've used an exponent? We have **logarithms** to help.

First, let's review. **Exponents** are simply an easy way to write repeated multiplication of some number. Expressions with exponents have a **base**, or the number being multiplied, and the exponent, or the number of times that base is being multiplied.

$$(\textbf{base})^{\textbf{exponent}}$$

For example:

$$2^2 = \underbrace{2 \cdot 2}_{\text{2 times}} = 4$$

$$5^3 = \underbrace{5 \cdot 5 \cdot 5}_{\text{3 times}} = 125$$

$$10^{21} = \underbrace{10 \cdot 10 \cdot \ldots\ldots \cdot 10 \cdot 10}_{\text{21 times}} = 1000000000000000000000$$

If you've ever heard of scientific notation (you probably used it in chemistry class), then you saw exponents with base 10, called **powers of 10**, to represent really big (or really small) numbers.

It seems pretty obvious how you'd make a really big number with exponents. If you just keep multiplying 10 by itself, it's going to get really big, really quickly!

Actually, the speed at which a number multiplied by itself grows has a special name. It is called **exponential growth** because the number grows so quickly.

For example, consider 10:

$10^0 = 1$

$10^1 = 10$

$10^2 = 10.10 = 100$

$10^3 = 10.10.10 = 1000$

$10^4 = 10.10.10.10 = 10000$

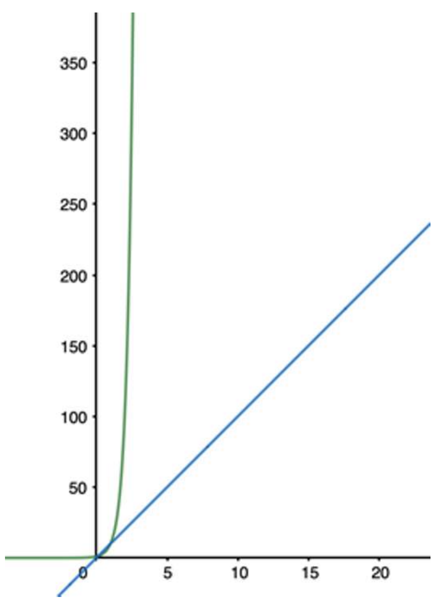
$10^5 = 10.10.10.10.10 = 100000$

$10^6 = 10.10.10.10.10.10 = 1000000$

.
.
.

10x

10^x is already equal to 1,000,000 when x = 6. (Compare that to 10x which is only equal to 60!)



Here's a picture showing you how quickly 10x grows. (The green line is y=10x and the blue line is y=10x.)

But if you dig up any old, hazy memories of Chemistry class, you might remember that you can make really *small* numbers using powers of 10 also. That's because a negative exponent is the same as a fraction. (What??)

$$10^{-2} = \frac{1}{10^2}$$
$$10^{-5} = \frac{1}{10^5}$$
$$\longrightarrow \quad 10^{-n} = \frac{1}{10^n}$$

When an exponent is negative, we just move it's base (and it) to the bottom of a fraction and make the exponent positive. Now we are *dividing* by a really huge number, which will give us a really small number.

Ok, enough about exponential growth. Back to Lebron James. (Really, back to logarithms.)

Unlike toothpaste, we can "undo" things in math. Exponents are no different. There is a function, called a **logarithm**, that takes the base and the "answer" for an exponential expressions, and gives you what the actual exponent was. (We can also call this exponent the **power**.)

$$(\text{base})^{\text{exponent}} = \text{answer} \xrightarrow{\text{"undo"}} \log_{\text{base}}(\text{answer}) = \text{exponent}$$

For example:

$$5^3 = 125 \longrightarrow \log_5(125) = 5$$

$$16^2 = 256 \longrightarrow \log_{16}(256) = 16$$

$$2^{-7} = \frac{1}{128} \longrightarrow \log_2\left(\frac{1}{128}\right) = -7$$

There are a couple of rules about logarithms though. The first rule is that the base number has to be positive. The second rule comes from the first - if the base number is always positive, then the "answer" will always be positive. Thus, the input for the log function has to be positive as well.

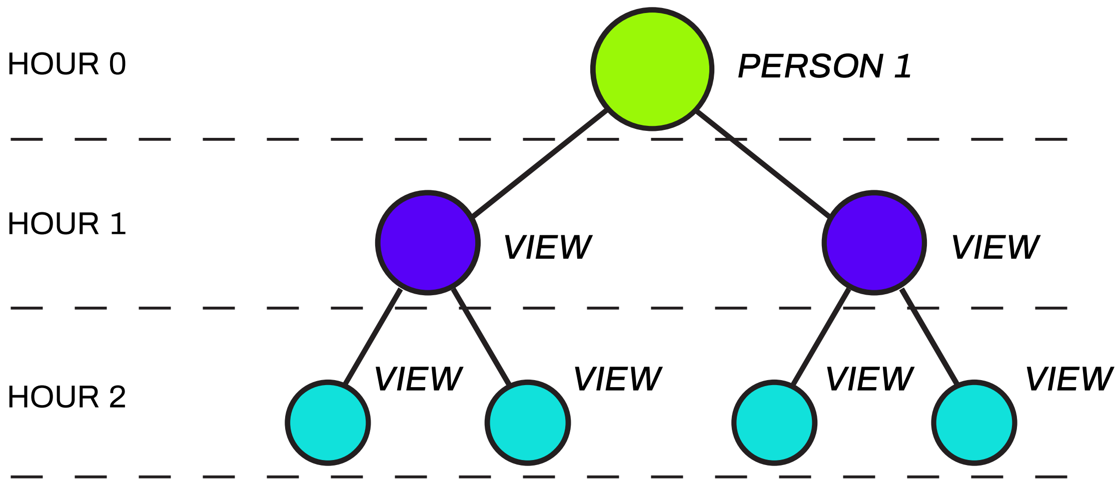
Actually, every log function has to have a base, and each log function with a unique base is… unique. Take a look at this graph of different log functions:

Viral Videos

You are a social media manager and want to reach as many people as possible with your new post. You have 12 hours to make your post go viral (getting at least 1000 views per hour).

Using your knowledge of exponents and logarithms, figure out how many views each person would need to generate to make your post viral by Hour 12 (you want the number of views to increase by at least 1000 between Hour 11 and 12).

For example, if each person who sees the post gets two other people to view it, that person generates two views. How many views would each person need to generate to reach viral status by Hour 12?



Write an equation using exponents to represent the growth of your post.

Then, using your equation, predict how many people will have viewed the post at 24 hours, 48 hours, and 72 hours.

Additionally, use your equation and knowledge of logarithms to determine at what hour the post will have 100,000 views.

S.T.E.A.M. Presentation

Create a presentation using exponents and logarithms to explain your "model" to go viral. Your presentation must show how you developed the equation (using exponents) to find the number of views each person needed to generate to go viral by hour 12, how you utilized your equation to predict views at hours 24, 48 and 72, AND how you used logarithms to find out what hour the post will have 100,000 views by.

On a separate sheet of paper, create five sections as shown below. Then, write a 5-paragraph essay analyzing how you developed the equation (using exponents) to find the number of views each person needed to generate to go viral by hour 12, how you utilized your equation to predict views at hours 24, 48 and 72, AND how you used logarithms to find out what hour the post will have 100,000 views by.

Paragraph 1: Summary
Use complete sentences to restate the project in your own words, identifying important information in the project. Use numbers with units in your description of any quantities.

Paragraph 2: Strategy
Use complete sentences and academic vocabulary to write the steps you would take to solve the problem. Do not use any numbers or computations in your description.

Paragraph 3: Solution
Use complete sentences, an organized presentation of mathematical computations (e.g. graphs, tables, equations, etc.), and your strategy to demonstrate the solution to the problem.

Paragraph 4: Justification
Use complete sentences and flexible problem solving strategies to construct viable arguments that demonstrate the accuracy of your solution.

Paragraph 5: Reflection
Use complete sentences and academic vocabulary to reflect on what you did well, what you did not do well, and what will you do differently next time to fix any errors.